division of Tortoises is confined to Aldabra and never extended to the Mascarenes proper, and that the Tortoises from the latter islands can be externally, though not osteologically, distinguished *as a whole* from the Galapagos Tortoises, as will be seen from the following synopsis :—

I. Nuchal plate absent. Frontal portion of the skull flat. Fourth cervical vertebra biconvex. Pelvis with broad symphysial bridge.

  A. Gular plate double; sternum of moderate extent .... GALAPAGOS TORTOISES.

  B. Gular plate single; sternum short ...... MASCARENE TORTOISES.

    a. Carapace thin, thickened towards the margins; centre of the last vertebral plate raised into a hump, which is separated from the penultimate vertebral by a transverse depression : *Tortoises of Mauritius* (*T. triserrata, T. inepta, T. indica, T. leptocnemis*).

    b. The entire carapace extremely thin and fragile, all the bones very slender: *Tortoise of Rodriguez* (*T. Vosmæri*).

II. Nuchal plate present. Frontal portion of the skull convex. Third cervical vertebra biconvex. Pelvis with narrow symphysial bridge. Gular plate double. Carapace thick. ALDABRA TORTOISES (*T. elephantina, T. Daudinii, T. ponderosa, T. hololissa*).

II. "On certain Definite Integrals." By W. H. L. RUSSELL, F.R.S. Received December 5, 1876.

The following paper is a continuation of one recently inserted in the 'Proceedings of the Royal Society.'

$$(13.)\quad \int_0^{\frac{\pi}{2}} d\theta \log_\epsilon (1+2\alpha \cos^2\theta \cos 2\theta+\alpha^2 \cos^4\theta)=\pi \log_\epsilon \frac{a+4}{4}.$$

$$(14.)\quad \int^{\infty} \frac{d\theta\sqrt{1+a\cos\theta+\sqrt{1+2a\cos\theta+\alpha^2}}}{1+\theta^2}=\frac{\pi}{\sqrt{2}}\cdot\frac{\sqrt{\epsilon-a}}{\sqrt{\epsilon}}.$$

Similarly we may find

$$(15.)\quad \int_0^{\infty} d\theta\sqrt{\cos 2\theta+\mu\cos 3\theta+\sqrt{1+\ 2\mu\cos\theta+\mu}}\log_\epsilon\frac{\theta^2+\zeta^2}{\theta^2+a^2}.$$

$$(16.)\quad \int_0^{\frac{\pi}{2}} \frac{\log_\epsilon \cos\theta\cos 2\theta}{1-2x^2\cos 4\theta+x^4}=\frac{\pi}{8x(1-x^2)}\log_\epsilon\frac{1+x}{1-x}.$$

$$(17.)\quad \int_0^{\pi} \theta d\theta\cdot\frac{\sin\theta+x^2\sin 3\theta}{1-2x^2\cos 4\theta+x^4}=\frac{\pi}{\sqrt{x}}\left\{\log_\epsilon\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)^{\frac{1}{4}}+\frac{1}{2}\tan^{-1}\sqrt{x}\right\}.$$

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(18.) $$\int_0^{\frac{\pi}{2}} \theta d\theta \frac{\sin\theta\cos\theta}{1+(2a-a^2)\cos^2\theta+a^2(a^2+2a+4)\cos^4\theta}=\frac{\pi}{(a^2+2a)\sqrt{3}}\tan^{-1}\frac{a\sqrt{3}}{a+4}.$$

(19.) $$\int_0^{\pi} \frac{\epsilon^{x\cos\theta}\big(\epsilon^{x\cos\theta}\sin\theta+\sin(\theta+x\sin\theta)\big)\theta d\theta}{\epsilon^{2x\cos\theta}+2\epsilon^{x\cos\theta}\cos(x\sin\theta)+1}=\frac{\pi}{x}\log_\epsilon\frac{2\epsilon^x}{\epsilon^x+1}.$$

(20.) $$\int_0^{\pi} \frac{\epsilon^{x\cos\theta}\big(\epsilon^{x\cos\theta}\sin\theta+\sin(\theta+x\sin\theta)\big)d\theta}{\epsilon^{2x\cos\theta}+2\epsilon^{x\cos\theta}\cos(x\sin\theta)+1}=1.$$

(21.) $$\int_0^{\frac{\pi}{2}} \cos\theta d\theta \frac{\cos(\theta+\tan\theta)+\alpha\cos\theta}{1+2\alpha\cos\theta+\alpha^2}=\frac{\pi}{4(\epsilon+\alpha)}.$$

(22.) $$\int_0^{\frac{\pi}{2}} \cos\theta d\theta \frac{\cos(\theta+c\tan\theta)+\alpha\cos\theta\cos c\tan\theta}{\sin^2\theta+(1+2a+a^2)\cos^2\theta}=\frac{\pi}{2(a+2)\epsilon^c}.$$

(23.) $$\int_0^{\frac{\pi}{2}} \frac{d\theta}{\cos\theta}\cdot\frac{\cos(\theta-\tan\theta)+\alpha\cos\theta\cos\tan\theta}{\sin^2\theta+(1+2a+a^2)\cos^2\theta}=\frac{\pi}{\epsilon^{\alpha+1}}.$$

(24.) $$\int_0^{\infty} \frac{dx(\epsilon^{x\cos\alpha}\cos\alpha\sin\alpha+\mu)}{\epsilon^{2x\cos\alpha}+2\mu\epsilon^{x\cos\alpha}\cos\alpha\sin\alpha+\mu^2}=\frac{\cos\alpha}{\mu}\log_\epsilon(1+\mu).$$

(25.) $$\int_0^{\infty} \frac{dx\epsilon^{x\cos\alpha}\sin(x\sin\alpha)}{\epsilon^{2x\cos\alpha}+2\mu\epsilon^{x\cos\alpha}\cos\alpha\sin\alpha+\mu^2}=\frac{\sin\alpha}{\mu}\log_\epsilon(1+\mu).$$

(26.) $$\int_0^{\infty} \frac{(\epsilon^{2x\cos\alpha}+\epsilon^{x\cos\alpha})\cos(x\sin\alpha)dx}{\epsilon^{4x\cos\alpha}+2\epsilon^{2x\cos\alpha}\cos(2x\sin\alpha)+1}=\frac{\pi}{4}\cos\alpha.$$

(27.) $$\int_0^{\infty} \frac{(\epsilon^{3x\cos\alpha}+\epsilon^{x\cos\alpha})\sin(x\sin\alpha)dx}{\epsilon^{4x\cos\alpha}+2\epsilon^{2x\cos\alpha}\cos(2x\sin\alpha)+1}=\frac{\pi}{4}\sin\alpha.$$

(28.) $$\int_0^{\pi} \theta d\theta\sin\theta\cos^{n-m-1}\theta\,\frac{1-\cos^{2n}\theta}{1-\cos^{2n}\theta}=\frac{\pi^2}{2n}\tan\frac{m\pi}{2n}.$$

(29.) $$\int_0^{\pi} \theta d\theta\sin\theta\cos^{n-m-1}\theta\,\frac{1-\cos^{2m}\theta}{1-\cos^{n}\theta}=\frac{\pi}{m}-\frac{\pi^2}{n}\operatorname{cotan}\frac{m\pi}{n}.$$

(30.) $$\int_0^{\frac{\pi}{2}} \frac{d\theta\cos^n\theta(\cos n\theta+\mu\cos^n\theta)}{(1+2\mu\cos^n\theta\cos n\theta+\mu^2\cos^{2n}\theta)(a^2\sin^2\theta+b^2\cos^2\theta)}=\frac{\pi}{2ab}\cdot\frac{a^n}{(a+b)^n+\mu a^n}.$$

$$(31.)\int_0^{\pi} d\theta\epsilon^{\sin 2\theta\cos 2\theta}\cos(\sin^2\theta\sin 2\theta)=\frac{\pi}{\sqrt[4]{\epsilon}}.$$

$$(32.)\int_0^{\pi} d\theta\sin\theta\epsilon^{\sin 2\theta\cos 2\theta}\sin(\sin^2\theta\sin 2\theta+\theta)=\frac{\pi}{2\sqrt[4]{\epsilon}}.$$

$$(33.)\int_0^{\frac{\pi}{2}} d\theta\theta\{\epsilon^{a\sin 2\theta}\sin 2(\theta-a\cos^2\theta)+\epsilon^{-a\sin 2\theta}\sin 2(\theta+a\cos^2\theta)\}=\frac{\pi\sin a}{2a}.$$

$$(34.)\int_0^{\omega}\frac{(x-a^2x)dx}{\big((1+a+a^2)\sin ax-a\sin 3ax\big)(1+x^2)}=\pi\frac{\epsilon^{2a}+a}{\epsilon^{2a}-a}\cdot\frac{\epsilon^{a}}{\epsilon^{2a}-1}.$$

$$(35.)\int_0^{\omega}\frac{(x-a^2x)dx}{\big((1+a+a^2)\cos ax+a\cos 3ax\big)(1+w^2)}=\pi\frac{\epsilon^{2a}-a}{\epsilon^{2a}+a}\cdot\frac{\epsilon^{a}}{\epsilon^{2a}+1}.$$

$$(36.)\int_0^{\pi} d\theta\epsilon^{x\cos\theta}\cos(x\sin\theta+\theta)\log_\epsilon(1+\cos\theta)=\frac{\pi(\epsilon^x-1)}{\epsilon^x x}.$$

$$(37.)\int_0^{\frac{\pi}{2}}\frac{d\theta\log\,(1+2a\cos^n\theta\cos n\theta+a^2\cos^{2n}\theta)}{a^2\sin^2\theta+b^2\cos^2\theta}=\frac{\pi}{ab}\log_\epsilon\frac{(a+b)^n+aa^n}{(a+b)^n}.$$

I observe that in (28) n and m must be odd and even or *vice versâ*, and that in (29) n must be even and m odd.

The integrals here are not solitary examples: each of them may be considered as the representative of a very numerous class. I have selected as examples those which seemed most suitable to be brought before the Royal Society.

P.S.—The following were added 13th February, 1877 :—

$$(38.)\int_0^{\pi}\theta d\theta\,\epsilon^{x\cos\theta}\sin\left(x\sin\theta+\frac{3\theta}{2}\right)\cos\frac{\theta}{2}=\frac{\pi}{2}\left(\frac{x-1}{x^2}+\frac{1}{\epsilon^x x^2}\right).$$

$$(39.)\int_0^{\pi}\theta d\theta\frac{\sin 2\theta+(1-x)\sin\theta}{1-2x\cos\theta+x^2}=\pi\left\{\frac{(x+1)\log_\epsilon(1+x)-x}{x^2}\right\}.$$